

A Quintessence Problem in Self-interacting Brans-Dicke Theory

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A quintessence scalar field in self-interacting Brans-Dicke theory is shown to give rise to a non-decelerated expansion of the present universe for open, flat and closed models. Along with providing a non-decelerating solution, it can potentially solve the flatness problem too.

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I. INTRODUCTION

The Standard Cosmological Model (SCM) can only describe decelerated universe models and so cannot reproduce the results coming from the recent type Ia supernovae observations upto about $z \sim 1$ [1] which favour an accelerated current universe. But as the SCM can give a satisfactory explanation to other observational properties of the present universe (e.g., primordial nucleosynthesis, extragalactic sources redshift, cosmic microwave radiation). The recent extensive search for a matter field which can give rise to an accelerated expansion for the universe. This type of matter field is called ‘quintessence matter’ (or, shortly Q-matter) This Q-matter can behave like a cosmological constant [2] by combining positive energy density and negative pressure. So there must be this Q-matter either neglected or unknown responsible for this accelerated universe. At the present epoch, a lot of works has been done to solve this quintessence problem and most popular candidates for Q-matter has so far been a scalar field having a potential which generates a sufficient negative pressure. The quintessence proposal faces two types of problems [3]. One of these problems (referred as fine tuning problem, eliminate classically by Ratra and Peebles [2]), is the smallness of the energy density compared to other typical particle physics scales. The other problem known as the cosmic coincidence problem is that although the missing energy density and matter density decrease at different rates as the universe expands, it appears that the initial condition has to be set so precisely that the two densities become comparable today. Quintessence has been proposed as that missing energy density component that along with the matter and baryonic density makes the density parameter equal to 1. A special form of quintessence field called the ‘tracker field’ has been proposed by Ratra and Peebles to tackle this problem [2] (see also ref. [4]). Since in a variety of inflationary models scalar fields have been used in describing the transition from the quasi-exponential expansion of the early universe to a power law expansion, it is natural to try to understand the present acceleration of the universe which has an exponential behaviour too, by constructing models where the matter responsible for such behaviour is also represented by a scalar field [5]. Inverse power law is the other potential [6] that has been studied extensively for quintessence models, particularly, for solving the cosmic coincidence problem. Recently, Bertolami and Martins [7] obtained an accelerated expansion for the universe in a modified Brans-Dicke (BD) theory by introducing a potential which is a function of BD scalar field itself. Very recently, Banerjee et al [8,9] also have shown that it is possible to have an accelerated universe with BD-theory in Friedmann model without any matter.

This paper investigates the possibility of obtaining a non-decelerating ($q \leq 0$) expansion for the universe in BD theory with scalar field which is minimally coupled to gravity and serves as the quintessence matter.

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II. FIELD EQUATIONS AND SOLUTIONS

The Brans-Dicke theory is given by the action [10]

$$S = \int \sqrt{-g} \left[\phi R - \frac{\omega}{\phi} \phi_{,\alpha} \phi^{,\alpha} + L_m \right] d^4x \quad (1)$$

where ϕ is the BD scalar field, ω is the dimensionless constant BD parameter and L_m is the Lagrangian for all other matter fields. We have chosen the units $8\pi G = c = 1$.

The matter content of the universe is composed of perfect fluid and a scalar field ψ as the quintessence matter. We assume that the universe is homogeneous and we consider an anisotropic space-time with line-element

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 d\Omega_k^2 \quad (2)$$

where a, b are functions of time only and

$$d\Omega_k^2 = \begin{cases} dy^2 + dz^2, & \text{when } k = 0 \text{ (Bianchi I model)} \\ d\theta^2 + \sin^2\theta d\phi^2, & \text{when } k = +1 \text{ (Kantowski-Sachs model)} \\ d\theta^2 + \sinh^2\theta d\phi^2, & \text{when } k = -1 \text{ (Bianchi III model)} \end{cases}$$

Here k is the spatial curvature index, so that the above three types [11] of models are Euclidean, closed and semi-closed respectively.

Now the BD-field equations are

$$2\frac{\ddot{b}}{b} + \left(\frac{\dot{b}}{b}\right)^2 + \frac{k}{b^2} = -\frac{(p_m + p_\psi)}{\phi} - \frac{1}{2}\omega \left(\frac{\dot{\phi}}{\phi}\right)^2 - 2\frac{\dot{b}}{b}\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi} \quad (3)$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} = -\frac{(p_m + p_\psi)}{\phi} - \frac{1}{2}\omega \left(\frac{\dot{\phi}}{\phi}\right)^2 - \left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right)\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi} \quad (4)$$

$$\left(\frac{\dot{b}}{b}\right)^2 + 2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{k}{b^2} = \frac{(\rho_m + \rho_\psi)}{\phi} - \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right)\frac{\dot{\phi}}{\phi} + \frac{\omega}{2} \left(\frac{\dot{\phi}}{\phi}\right)^2 \quad (5)$$

and the wave equation for the BD scalar field ϕ is

$$\ddot{\phi} + \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}\right)\dot{\phi} = \frac{1}{3+2\omega} [(\rho_m - 3p_m) + (\rho_\psi - 3p_\psi)] \quad (6)$$

ρ_m and p_m are the density and the pressure of normal matter, ρ_ψ and p_ψ are those due to the quintessence field given by

$$\rho_\psi = \frac{1}{2}\dot{\psi}^2 + V(\psi), \quad p_\psi = \frac{1}{2}\dot{\psi}^2 - V(\psi) \quad (7)$$

where $V(\psi)$ is the relevant potential.

The wave equation for the quintessence scalar field ψ is

$$\ddot{\psi} + \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \right) \dot{\psi} = -\frac{dV(\psi)}{d\psi} \quad (8)$$

From the above field equations, we have the matter ‘conservation’ equation

$$\dot{\rho}_m + \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \right) (\rho_m + p_m) = 0 \quad (9)$$

Assuming that at the present epoch, the universe is filled with cold matter (dust) with negligible pressure, so using $p_m = 0$, the conservation equation (9) gives

$$\rho_m = \frac{\rho_1}{ab^2} \quad (10)$$

where ρ_1 is an integration constant.

Now we assume, the power law form of scale factors a, b and the BD scalar field ϕ are

$$a = a_1 t^\alpha, \quad b = b_1 t^\beta, \quad \phi = \phi_1 t^\delta \quad (11)$$

where a_1, b_1, ϕ_1 are positive constants and α, β, δ are real constants with $\alpha + 2\beta \geq 3$ (for accelerating universe).

From the field equations (3), (4) and (5) using (11) we have the expression for $\dot{\psi}^2$ as

$$\dot{\psi}^2 = \frac{2k\phi_1}{b_1^2} t^{\delta-2\beta} - \frac{\rho_1}{a_1 b_1^2} t^{-\alpha-2\beta} + \phi_1 [2\beta^2 - 2\alpha(\alpha-1) - \omega\delta^2 - \delta(\delta-1) - (\alpha-2\beta)\delta] t^{\delta-2} \quad (12)$$

From (6) the potential V is given by

$$V = \frac{k\phi_1}{2b_1^2} t^{\delta-2\beta} - \frac{\rho_1}{2a_1 b_1^2} t^{-\alpha-2\beta} + \frac{1}{4} \phi_1 [(2\omega+3)(\alpha+2\beta+\delta-1)\delta + 2\beta^2 - 2\alpha(\alpha-1) - \omega\delta^2 - \delta(\delta-1) - (\alpha-2\beta)\delta] t^{\delta-2} \quad (13)$$

The wave equation (8) for the quintessence scalar field ψ can be written in the form

$$-\frac{dV}{dt} = \dot{\psi} \ddot{\psi} + \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \right) \dot{\psi}^2 \quad (14)$$

After integration (14) the expression for V is

$$V = -\frac{k\phi_1}{b_1^2} \frac{(2\alpha+2\beta+\delta)}{(\delta-2\beta)} t^{\delta-2\beta} - \frac{\rho_1}{2a_1 b_1^2} t^{-\alpha-2\beta} + \frac{\phi_1(2\alpha+4\beta+\delta-2)}{2(\delta-2)} [2\alpha(\alpha-1) + \delta(\delta-1) + \omega\delta^2 + (\alpha-2\beta)\delta - 2\beta^2] t^{\delta-2} \quad (15)$$

Now, the consistency relations of the constants coming from the two identical equations (13) and (15) for the potential V are

$$4\alpha + 2\beta + 3\delta = 0 \quad (16)$$

$$(2\omega + 3)\delta(\delta - 2)(\alpha + 2\beta + \delta - 1) = (4\alpha + 8\beta + 3\delta - 6)[2\alpha(\alpha - 1) + \delta(\delta - 1) + \omega\delta^2 + (\alpha - 2)\delta - 2\beta^2] \quad (17)$$

From these relations for an accelerating universe ($\alpha + 2\beta \geq 3$), there are two possibilities: one in which $k = 0$ and the second where $k \neq 0$.

Case I : $k = 0$

In this case , consistency conditions are (17) and

$$\alpha + 2\beta = c \quad (18)$$

where c is an constant (≥ 3).

For solving (17) and (18), we may choose $\delta = -2$ and we have two possible solutions:

$$(i) \quad \alpha = \beta = \frac{c}{3}, \quad \delta = -2$$

or

$$(ii) \quad \alpha = 4 - c, \quad \beta = -2 + c, \quad \delta = -2$$

For both solutions (i) and (ii) the expression for $\dot{\psi}^2$ (see eq.(12)) becomes

$$\dot{\psi}^2 = -\frac{\rho_1}{a_1 b_1^2} t^{-c} - 2(2\omega + 3)\phi_1 t^{-4} \quad (19)$$

This indicates that $\omega < -3/2$ as $\dot{\psi}^2$ cannot be negative.

For $c = 4$, one has $2|2\omega + 3|\phi_1 \geq \frac{\rho_1}{a_1 b_1^2}$ and the deceleration parameter $q = -\frac{1}{4}$.

In this case, equation (19) integrates to

$$\psi = \pm \frac{A}{t} \quad (20)$$

where $A^2 = -2(2\omega + 3)\phi_1 - \frac{\rho_1}{a_1 b_1^2}$ and the relation between V and ψ becomes

$$V = V_1 \psi^4 \quad (21)$$

where V_1 being a constant, related to the constants a_1, ρ_1 etc. The model works for all time $0 < t < \infty$ with the condition $2|2\omega + 3|\phi_1 \geq \frac{\rho_1}{a_1 b_1^2}$ is satisfied. The value of ω is related to the other constants as follows:

$$2(2\omega + 3)\phi_1 + \frac{\rho_1}{a_1 b_1^2} = \frac{-1 \pm \sqrt{1 + \frac{64}{9} V_1}}{4V_1}, \quad \text{for solution (i)}$$

and

$$2(2\omega + 3)\phi_1 = \frac{\rho_1}{a_1 b_1^2} - \frac{\epsilon}{2V_1}, \quad (\epsilon = 0, 1), \quad \text{for solution (ii)}$$

For $c \neq 4$, the model does not work for the whole range of time $0 < t < \infty$. In this case, the deceleration parameter is $q = \frac{3}{c} - 1$.

If $c > 4$, then the rate of acceleration is faster than $q = -\frac{1}{4}$ and from equation (19), $\dot{\psi}^2 > 0$ restricts the validity of the model for $t > t_1$ where

$$t_1 = \left[\frac{\rho_1}{2|2\omega+3|a_1b_1^2\phi_1} \right]^{\frac{1}{c-4}} \quad (21a)$$

Further if $3 < c < 4$, then the universe will expands with an acceleration but with a rate less than $q = -\frac{1}{4}$ and as before from equation (19) for real $\dot{\psi}$, the model works upto the time t_2 where

$$t_2 = \left[\frac{\rho_1}{2|2\omega+3|a_1b_1^2\phi_1} \right]^{\frac{1}{c-4}} \quad (21b)$$

For $q = -\frac{1}{4}$, the present age of the universe can be calculated from (5) as

$$t_0 = \left[2 - 2\omega - \frac{A^2}{2\phi_1} + \frac{V_1 A^4}{\phi_1} \right]^{1/2} \frac{1}{[(H_b^2)_0 + 2(H_a)_0(H_b)_0]^{1/2}}$$

where $H_a = \frac{\dot{a}}{a}$ and $H_b = \frac{\dot{b}}{b}$.

For large ω limit,

$$t_0 \cong \frac{\sqrt{-2\omega}}{[(H_b^2)_0 + 2(H_a)_0(H_b)_0]^{1/2}}$$

where ω is obviously a negative quantity.

Case II : $k \neq 0$.

In this case, consistency conditions are (16), (17) and

$$\alpha + 2\beta = 3 \quad (22)$$

Solving these three equations we have the following solutions:

(i) $\alpha = 1, \beta = 1, \delta = -2$ for all values of ω .

(ii) $\alpha = 1 \mp \sqrt{\frac{2\omega+3}{2\omega-3}}, \beta = 1 \pm \sqrt{\frac{2\omega+3}{2\omega-3}}, \delta = -2 \pm \sqrt{\frac{2\omega+3}{2\omega-3}}$

provided for $\omega > 3/2$ or $\omega \leq -3/2$.

For the solution (i), the model works for a limited period of time, $0 < t < t_1$ where

$$t_1 = 2\phi_1 \left[\frac{k}{b_1^2} - (2\omega + 3) \right] \frac{a_1 b_1^2}{\rho_1} \quad (23)$$

For an open universe i.e., for $k = -1, (2\omega + 3) < 0$ and $|2\omega + 3| > \frac{1}{b_1^2}$.

For a closed universe i.e., for $k = +1, (2\omega + 3) < 0$ and $|2\omega + 3| < \frac{1}{b_1^2}$.

For the solution (ii) the model works for the time, where t satisfies the equation

$$2(2\omega + 3)\phi_1 \left[\left(1 \mp \sqrt{\frac{2\omega + 3}{2\omega - 3}} \right)^2 - \frac{2}{2\omega - 3} \right] t^{\pm 2\sqrt{\frac{2\omega + 3}{2\omega - 3}}} + \frac{\rho_1}{a_1 b_1^2} t \leq \frac{2k\phi_1}{b_1^2} \quad (24)$$

provided for $\omega > 3/2$ or $\omega \leq -3/2$.

III. FLATNESS PROBLEMS AND ITS SOLUTIONS

One important aspect of this model is that potentially it can solve the flatness problem. Now we make a conformal transformation [12] as

$$\bar{g}_{\mu\nu} = \phi g_{\mu\nu} \quad (25)$$

In this section, we make the following transformations:

$$\begin{aligned} d\eta = \sqrt{\phi} a, \quad \bar{a} = \sqrt{\phi} a, \quad \bar{b} = \sqrt{\phi} b, \quad \psi = \ln \phi, \quad \bar{\rho}_m = \phi^{-2} \rho_m, \quad \bar{\rho}_\psi = \phi^{-2} \rho_\psi, \\ \bar{p}_m = \phi^{-2} p_m, \quad \bar{p}_\psi = \phi^{-2} p_\psi \end{aligned} \quad (26)$$

So the field equations (3) - (5) transformed to (after some manipulations)

$$\left(\frac{\bar{b}'}{\bar{b}}\right)^2 - 2\frac{\bar{a}''}{\bar{a}} - 2\frac{\bar{a}'}{\bar{a}}\frac{\bar{b}'}{\bar{b}} + \frac{k}{\bar{b}^2} = (\bar{p}_m + \bar{p}_\psi) + \frac{(3+2\omega)}{4} \left(\frac{\phi'}{\phi}\right)^2 \quad (27)$$

$$\left(\frac{\bar{b}'}{\bar{b}}\right)^2 + 2\frac{\bar{a}'}{\bar{a}}\frac{\bar{b}'}{\bar{b}} + \frac{k}{\bar{b}^2} = (\bar{\rho}_m + \bar{\rho}_\psi) + \frac{(3+2\omega)}{4} \left(\frac{\phi'}{\phi}\right)^2 \quad (28)$$

and

$$\frac{\bar{b}''}{\bar{b}} - \frac{\bar{a}''}{\bar{a}} + \frac{k}{\bar{b}^2} = \frac{\bar{a}'}{\bar{a}}\frac{\bar{b}'}{\bar{b}} - \left(\frac{\bar{b}'}{\bar{b}}\right)^2 \quad (29)$$

where $' \equiv \frac{d}{d\eta}$.

The BD scalar field in new version $\bar{\rho}_\phi$ is given by

$$\bar{\rho}_\phi = \frac{(3+2\omega)}{4} \left(\frac{\phi'}{\phi}\right)^2 = \bar{p}_\phi \quad (30)$$

We define the dimensionless density parameter $\bar{\Omega}$ as

$$\bar{\Omega} = \bar{\Omega}_m + \bar{\Omega}_\phi + \bar{\Omega}_\psi = \frac{\bar{\rho}}{3\bar{H}^2} \quad (31)$$

where $\bar{\rho} = \bar{\rho}_m + \bar{\rho}_\phi + \bar{\rho}_\psi$ is the total density and $\bar{\Omega}_i$ are defined accordingly.

Using (27)-(30), and combining the energy densities, we have the equation for the conservation for the total energy,

$$\bar{\rho}' + 3\bar{H}(\bar{\rho} + \bar{p}) = 0 \quad (32)$$

Here $\bar{H} = \frac{1}{3} \left(\frac{\bar{a}'}{\bar{a}} + 2\frac{\bar{b}'}{\bar{b}}\right)$ is the Hubble parameter in the Einstein frame and γ is the net barotropic index defined as

$$\gamma \bar{\Omega} = \gamma_m \bar{\Omega}_m + \gamma_\phi \bar{\Omega}_\phi + \gamma_\psi \bar{\Omega}_\psi \quad (33)$$

From equations (28) and (32), we have the evolution equation for the density parameter as

$$\bar{\Omega}' = \bar{\Omega}(\bar{\Omega} - 1)[\gamma\bar{H}_a + 2(\gamma - 1)\bar{H}_b] \quad (34)$$

where $\bar{H}_a = \frac{\bar{a}'}{\bar{a}}$ and $\bar{H}_b = \frac{\bar{b}'}{\bar{b}}$.

The individual γ_i 's are defined by the relation $p_i = (\gamma_i - 1)\rho_i$. So the ratios $\frac{p_i}{\rho_i}$ remain same in both frames. For our choices of matter, \bar{p}_m and $\bar{p}_\psi = \bar{\rho}_\psi$, so we have $\gamma_m = 1$ and $\gamma_\phi = 2$. The other index γ_ψ is related by the equation

$$\gamma_\psi = \frac{p_\psi + \rho_\psi}{\rho_\psi} = \frac{\dot{\psi}^2}{\frac{1}{2}\dot{\psi}^2 + V} \quad (35)$$

It has been shown that γ_ψ is varies with time. The equation (34) indicates that $\bar{\Omega} = 1$ is a possible solution and this solution determines that $\left(\frac{\partial \bar{\Omega}}{\partial \bar{\Omega}}\right)_{\bar{H}} < 0$. This solution is stable for expanding universe ($\bar{H} > 0$) if $\gamma < \frac{2}{3}$ with the relevant condition

$$\bar{\Omega}_m + 4\bar{\Omega}_\phi < (2 - 3\gamma_\psi)\bar{\Omega}_\psi \quad (36)$$

From the field equation (28), the curvature parameter $\bar{\Omega}_k = -k/\bar{b}^2$ vanishes for the solution $\bar{\Omega} = 1$. So for BD-scalar field it is possible to have a stable solution corresponding to $\bar{\Omega} = 1$ and hence the flatness problem can be solved.

IV. CONCLUDING REMARKS

In this work, we have investigated the nature of the potential relevant to the power law expansion of the universe in a self-interacting Brans-Dicke (BD) cosmology with a perfect fluid distribution for anisotropic cosmological models. We have considered a non-gravitational quintessence scalar field ψ with a potential $V = V(\psi)$. This scalar field in BD-theory is shown to give rise to an accelerated expansion for the present dust universe (where we have taken $p_m = 0$) It is to be noted that at early stages of the evolution of the universe p_m is non-zero. But if we take barotropic equation of state $p_m = (\gamma - 1)\rho_m$, then equation (10) is modified to $\rho_m = \frac{\rho_1}{(ab^2)^\gamma}$.

We have presented a class of solutions describing non-decelerating universe for both flat ($k = 0$) and curved space-time ($k \neq 0$). For $k = 0$, there are two possible solutions for different choice of the parameters. In both the solutions, the parameter ω must be negative (in fact $2\omega + 3$ is negative) to make the quintessence field real. The validity (on the time scale) of the solutions depends on the parameter c (defined in eq.(18)). For $c = 4$, the model works for all time $0 < t < \infty$, while for $c > 4$, the model works for (t_1, ∞) where t_1 is given by (21a). From this model we cannot predict the geometry of the universe before t_1 . Similarly for $3 < c < 4$ the model is valid for $t < t_2$ i.e., from the beginning to the time t_2 . Here also we cannot predict the nature of space-time of the universe after $t = t_2$.

In curved space-time ($k \neq 0$) there are also two possible solutions for different choice of the parameters. The time interval over which the solutions are valid are given in equations (23) and (24). For close model ($k > 0$) the coupling parameter ω is restricted by the inequality $0 < 2\omega + 3 < 1/b_1^2$ while for open model ω satisfies $-1/b_1^2 < 2\omega + 3 < 0$. From the above solutions, we note that the coupling parameter ω may be positive or negative (with some restrictions). Hence for all solutions the choice of ω is not in agreement with the observations.

Finally, for non-decelerating solution it can also potentially solve the flatness problem (without any restriction on the parameters) and it has been shown that $\bar{\Omega} = 1$ could be a stable solution in this model.

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